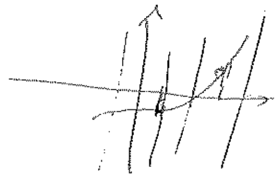
RAÍCES ECUACIONES NO LINEALES

$f(x)=0$



Bisección

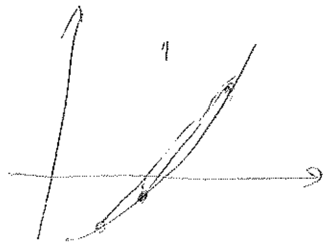
$a \rightarrow b$

$c = \dfrac{a+b}{2}$ → $f(a)\cdot f(c) > 0 \rightarrow b = c$; $f(a)\cdot f(c) > 0 \rightarrow a = c$

$(\ldots) \rightarrow |a-b| < \epsilon$

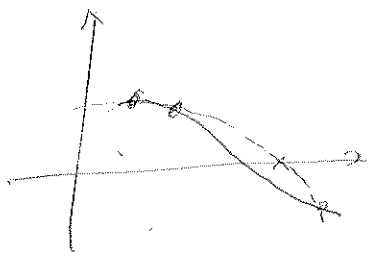
$|b-a| < \epsilon_{rel} \cdot MAX\left(\frac{a+b}{2}, |b-a|_0\right)$

$\dfrac{|b-a|}{|\alpha|} = \epsilon_{rel}$



Regula falsi

$\underbrace{c = \dfrac{a\, f(b) - b\, f(a)}{f(b) - f(a)}}_{\text{Secante: } x_{r+1} =}$ → + horquillado



Müller

x_0, x_1, x_2

$\rightarrow P(x) = a\,(x-x_2)^2 + b(x-x_2) + c$

$$a = \frac{(x_1-x_2)[f(x_0)-f(x_2)] - (x_0-x_2)[f(x_1)-f(x_2)]}{(x_0-x_2)(x_1-x_2)(x_0-x_1)}$$

$$b = \frac{(x_0-x_2)^2[f(x_1)-f(x_2)] - (x_1-x_2)^2[f(x_0)-f(x_2)]}{(x_0-x_2)(x_1-x_2)(x_0-x_1)}$$

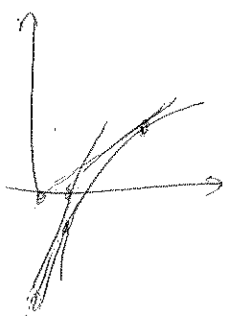
$c = f(x_2)$

$\rightarrow x_0 \leftarrow x_1, \; x_1 \leftarrow x_2, \; x_2 \leftarrow x_3$

$x_3 = x_2 + \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow$ menor $|\;|$

Newton - Raphson

$x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$



PROBLEMAS LINEALES

LU

$(B \cdot C = A)$

$$c_{ij} = a_{ij} - \sum_{k=1}^{i-1} b_{ik} c_{kj} \qquad i \leq j$$

$$b_{ij} = \frac{a_{ij} - \sum_{k=1}^{j-1} b_{ik} c_{kj}}{c_{jj}} \qquad i > j$$

$$\text{pivote} = \text{MAX}\{|c_{jj}|, |b'_{j+1\,j}|, \dots, |b'_{Nj}|\} \rightarrow \text{fila } j \times k(\to)$$

$$b'_{ij} = a_{ij} - \sum_{k=1}^{j-1} b_{ik} c_{kj}$$

+ guardar permutaciones

$$(3,1,2) \rightarrow P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad PA = LU$$

Determinante

$$|A| = (-1)^{\#\text{ permutas}} \cdot \prod_{k=1}^{N} c_{kk}$$

Sistema ecuas lineales

$$A \cdot \vec{x} = \vec{w} \longrightarrow P^{-1} L U \cdot \vec{x} = \vec{w}$$

$$\hookrightarrow L \cdot \underbrace{U \cdot \vec{x}}_{\vec{y}} = P^{-1} \cdot \vec{w} \rightarrow y_i = w_i' - \sum_{j=1}^{i-1} b_{ij} y_j \qquad \text{x orden, recurrente}$$

$$\hookrightarrow x_i = \frac{y_i - \sum_{j=i+1}^{N} c_{ij} x_j}{c_{ii}} \qquad \text{orden inverso}$$

Inversión matriz

$$A^{-1} = U^{-1} L^{-1} P$$

$$U^{-1} \longrightarrow e_{jj} = \frac{1}{c_{jj}}, \quad e_{ij} = -\frac{\sum_{k=i}^{j-1} e_{ik} c_{kj}}{c_{jj}} \qquad j > i$$

$$e_{12} = -\frac{e_{11} c_{12}}{c_{22}}$$

$$e_{13} = -\frac{e_{11} c_{13} + e_{12} c_{23}}{c_{33}}$$

$$e_{23} = - \frac{\dots + e_{22} c_{23}}{c_{33}}$$

$$L^{-1} \longrightarrow d_{ij} = \cancel{b_{ij} + \dots} - b_{ij} - \sum_{j+1}^{i-1} b_{ik} d_{kj} \qquad i > j$$

$$d_{21} = -b_{21}$$

$$d_{31} = -b_{31} - b_{32} \cdot d_{21}$$

$$d_{32} = -b_{32}$$

Descomposición Cholesky

Matrices simétricas + positivas

$A = L \cdot L^T$ — L triangular inferior

$$l_{ii} = \left[a_{ii} - \sum_{k=1}^{i-1} l_{ik}^2\right]^{1/2}$$

$$l_{ji} = \frac{1}{l_{ii}}\left[a_{ji} - \sum_{k=1}^{i-1} l_{jk} l_{ik}\right] \qquad j > i$$

$$l_{11} = \sqrt{a_{11}}$$

$$l_{21} = \frac{a_{21}}{l_{11}}$$

$$l_{31} = \frac{a_{31}}{l_{11}}$$

$$l_{22} = \sqrt{a_{22} - l_{21}^2}$$

$$l_{32} = \frac{1}{l_{22}}(a_{32} - l_{31} l_{21})$$

$$l_{33} = \sqrt{a_{33} - l_{31}^2 - l_{32}^2}$$

VALORES Y VECTORES PROPIOS

Método Jacobi

Matrices simétricas $A = A^T$

$$A \to \overbrace{R_1^T A R_1}^{A'} \to R_2^T R_1^T A R_1 R_2 \to \dots$$

$$R_{pq} = \begin{array}{c} \\ p \to \\ q \to \end{array} \begin{pmatrix} 1 & p & q & \\ \vdots & \cos\theta & \sin\theta & \vdots \\ \vdots & -\sin\theta & \cos\theta & \vdots \\ \vdots & \vdots & \vdots & \end{pmatrix}$$

$R_{pp} = \cos\theta = R_{qq}$

$R_{pq} = \sin\theta = -R_{qp}$

$R_{ii} = 1 \quad i \neq q, \neq p$

resto $= 0$

Eliminas a_{pq} !!>

$$\hookrightarrow \operatorname{tg} 2\theta = -\frac{2a_{pq}}{(a_{pp} - a_{qq})} \longrightarrow A' = R^T A R$$

$a'_{pq} = 0$

$a'_{ip} = a_{ip}\cos\theta - a_{iq}\sin\theta \qquad i \neq p, q$

$a'_{iq} = a_{ip}\sin\theta + a_{iq}\cos\theta \qquad i \neq p, q$

$a'_{pp} = a_{pp}\cos^2\theta + a_{qq}\operatorname{sen}^2\theta - 2a_{pq}\operatorname{sen}\theta\cos\theta$

$a'_{qq} = a_{pp}\operatorname{sen}^2\theta + a_{qq}\cos^2\theta + 2a_{pq}\operatorname{sen}\theta\cos\theta$

LU 3x3

$c_{11} = a_{11}$ $\qquad c_{12} = a_{12}$ $\qquad c_{13} = a_{13}$

$b_{21} = \dfrac{a_{21}}{c_{11}}$ $\qquad c_{22} = a_{22} - b_{21}c_{12}$ $\qquad c_{23} = a_{23} - b_{21}c_{13}$

$b_{31} = \dfrac{a_{31}}{c_{11}}$ $\qquad b_{32} = \dfrac{a_{32} - b_{31}c_{12}}{c_{22}}$ $\qquad c_{33} = a_{33} - b_{31}c_{13} - b_{32}c_{23}$

= s! kjjjjo!

$y_1 = w_1$

$y_2 = w_2 - b_{21}y_1$

$y_3 = w_3 - b_{31}y_1 - b_{32}y_2$

$x_3 = y_3 / c_{33}$

$x_2 = \dfrac{y_2 - c_{23}x_3}{c_{22}}$

$x_1 = \dfrac{y_1 - c_{12}x_2 - c_{13}x_3}{c_{11}}$

INTERPOLACIÓN

Polinomio interpolador

$$\mathcal{E}(x) = \frac{f^{n+1}(\xi(x))}{(n+1)!} \cdot \prod_{i=0}^{n} (x - x_i) \qquad \text{orden } n$$

Lagrange

$$P_n(x) = \sum_{k=0}^{n} \prod_{i=0}^{n} \frac{x - x_i}{x_k - x_i} y_k \qquad L_k(x_i) = \delta_{ik} \quad i = 1, \dots n$$

Diferencias divididas

$$f[x_i] = f(x_i)$$

$$\frac{f[x_{i+1}] - f[x_i]}{x_{i+1} - x_i} = f[x_i, x_{i+1}]$$

$$f[x_i, \dots x_{i+k}] = \frac{f[x_{i+1}, \dots x_{i+k}] - f[x_i, \dots x_{i+k-1}]}{x_{i+k} - x_i}$$

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, \dots x_k] \cdot \prod_{i=0}^{k-1} (x - x_i) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + \dots$$

Neville

$$P_0 = f_0$$

$$P_{i(i+1)(i+2)\dots(i+j)}(x) = \frac{(x - x_i) P_{(i+1)\dots(i+j)}(x) - (x - x_{i+j}) P_{i(i+1)\dots(i+j-1)}}{x_{i+j} - x_i}$$

Estimación error

$$\epsilon = \text{MAX}\{|P_{0123} - P_{012}|, |P_{0123} - P_{123}|\}$$

Chebysev

$$T_n(x) = \cos(n \arccos(x)) \qquad x \in [-1, 1] \quad \rightarrow x' = \frac{b-a}{2} x + \frac{b+a}{2}$$

$\hookrightarrow$ 0s

$$x = \cos\left(\frac{2k+1}{2n} \pi\right) \qquad n \rightarrow \text{nº puntos}$$

$$T_{n+1} = 2x\, T_n(x) - T_{n-1}(x) \qquad T_0(x) = 1, \quad T_1(x) = x$$

Splines

cúbicos: $S_j = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3$ $\qquad a_j = y_j$

- $S(x_j) = y_j$
- $S_{j+1}(x_{j+1}) = S_j(x_{j+1})$
- $S'_{j+1}(x_{j+1}) = S'_j(x_{j+1})$
- $S''_{j+1}(x_{j+1}) = S''_j(x_{j+1})$

$+$ $\hookrightarrow$ libre $\rightarrow$ $S''_0(x_0) = S''_n(x_n) = 0$

$\hookrightarrow$ sujeta $\rightarrow$ $S''(x_0) = f''(x_0)$; $S'(x_0) = f'(x_0)$

splines naturales
$S''_0(x_0) = S''_{n-1}(x_n) = 0$

sujetos
$S'_0(x_0) = f'_0(x_0)$; $S'_{n-1}(x_n) = f'_n$

$$\begin{pmatrix} 1 & 0 & \dots & & \\ h_0 & 2(h_0 + h_1) & h_1 & & \\ 0 & h_1 & 2(h_1 + h_2) & h_2 & \\ & & & \ddots & \\ & & & 0 & 1 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{3}{h_1}(a_2 - a_1) - \frac{3}{h_0}(a_1 - a_0) \\ \frac{3}{h_2}(3 - 2) - \frac{3}{h_1}(2 - 1) \\ \vdots \\ 0 \end{pmatrix}$$

$$d_j = \frac{c_{j+1} - c_j}{3h_j}$$

$$b_j = \frac{a_{j+1} - a_j}{h_j} - \frac{h_j}{3}(2c_j + c_{j+1})$$

n+1 puntos
orden n splines

DERIVACIÓN E INTEGRACIÓN NUMÉRICA

Derivadas

$$f'(x) = \frac{f(x+h) - f(x)}{h} - f''(\xi)\frac{h}{2!}$$

$$f'(x) = \frac{1}{2h}\left(f(x+h) - f(x-h)\right) - \frac{h^2}{6!} f'''(\xi)$$

$$f''(x) = \frac{1}{h^2}\left[f(x+h) + f(x-h) - 2f(x)\right] - \frac{h^2}{12} \cdot f^{iv}(\xi)$$

Richardson

$$D = d_0(h) + a_2 h^2 + a_4 h^4 + \dots = d_0\left(\tfrac{h}{2}\right) + \frac{a_2}{4} h^2 + \dots$$

$$= \frac{4 d_0\left(\frac{h}{2}\right) - d_0(h)}{3} - \frac{a_4}{4} h^4 + \dots$$

2^n intervalos

$$D(n,K) = \frac{4^K D(n,K-1) - D(n-1,K-1)}{4^K - 1}$$

n: 2^n int. ; K: orden extrap.

Integrales

$$\int_a^b f(x)\,dx \simeq h \sum_{K=0}^{n} \alpha_K f_K \qquad \alpha_K = \int_0^n \phi_K(s)\,ds \qquad \phi_K(s) = \prod_{i=0}^{n} \left(\frac{s-i}{K-i}\right)$$

• Trapezoidal

$$\frac{h}{2}(f_0 + f_1) \qquad \epsilon = \frac{h^3}{12} f''(\xi) \qquad \xrightarrow{\text{repetido}} \quad \frac{h}{2}[f_0 + f_n] + h\sum_{i=1}^{n-1} f_i$$

2^n intervalos

$$R(n) = \frac{R(n-1)}{2} + h \sum_{K=1}^{2^{n-1}} f(a + (2K-1)h), \quad R(0) = \left(f(a) + f(b)\right)\frac{(b-a)}{2}$$

$$\epsilon = \frac{b-a}{12} h^2 f''(\xi)$$

$$|R(n) - R(n-1)| < \epsilon$$

• Simpson

(3/8, Bode)

$$\int_a^b f(x)\,dx = \frac{h}{3}(f_0 + 4f_1 + f_2) - \frac{h^5}{90} f^{4}(\xi)$$

↳ Repetido: $= \frac{h}{3}\left(f_0 + f_n + 4\sum_{i=1}^{n/2} f(x_{2i-1}) + 2\sum_{i=2}^{n/2} f(x_{2i-2})\right) - \frac{1}{180}(b-a)h^4 f^{iv}(\xi)$

• Romberg

$$I \simeq R(n,0) + c_2 h^2 + c_4 h^4 + \dots$$

2^n intervalos → extrapola Richardson

$$R(n,m) = \frac{4^m R(n,m-1) - R(n-1,m-1)}{4^m - 1}$$

MODELADO DE DATOS EXPERIMENTALES

$$\chi^2 = \sum_{i=1}^{n} \frac{1}{\sigma_i^2}\left[y_i - f(x_i; p_1, \dots, p_m)\right] \rightarrow \frac{\partial \chi^2}{\partial p_k} \text{ mínimo}$$

$$\{x_i, y_i, \sigma_i\}_{i=1}^{n} \rightarrow f(\vec{x}, \vec{p}) \;/\; \vec{p} = \{p_1 \dots p_m\} \rightarrow \text{PMV}$$

Lineales en parámetros

$$f(x, \bar{p}) = \sum_{j=1}^{m} p_j f_j(x)$$

$$\rightarrow \underset{m\times m}{\mathbb{F}} \cdot \underset{m}{\vec{p}} = \vec{v}$$

$$\boxed{F_{kj} = F_{jk} = \sum_{i=1}^{n} \frac{f_k(x_i) f_j(x_i)}{\sigma_i^2} \qquad v_k = \sum_{i=1}^{n} y_i \frac{f_k(x_i)}{\sigma_i^2}}$$

- $\mathbb{F}$
- $\vec{v}$
- $p_j = \sum C_{jk} v_k$
- $\sigma(p_j) = \sqrt{C_{jj}}$
- χ^2

$\mathbb{F}^{-1} = C \equiv$ matriz covarianza $\rightarrow \boxed{\sigma^2(p_j) = C_{jj}}$

Recta

$p_1 + p_2 x \rightarrow$

$$p_1 = \frac{1}{|F|}\left\{\sum \frac{x_i^2}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2}\right\}$$

$$p_2 = \frac{1}{|F|}\left\{\sum \frac{1}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2}\right\}$$

Calidad ajustes

$\{x_i, y_i, \sigma_i\} \rightarrow N$ experimentos $\rightarrow N \times n$ puntos $\rightarrow N$ ajustes

N ~~x~~ m-pletes $\{p_1, \dots p_m\}$

N χ^2

$$\boxed{\chi^2 = \sum \frac{(y_i - f_i)^2}{\sigma_i^2} \cong \nu = n - m}$$

$\rightarrow$ Exp. $\rightarrow \chi_0^2 \approx \nu$

1) Si $\chi_0^2 \;/\; \nu - 2\sqrt{2\nu} \leq \chi_0^2 \leq \nu + \sqrt{2\nu}$ ✓

2) $\chi_0^2 << \nu \Rightarrow \frac{\chi_0^2}{\nu} \ll 1$ → errores muy grandes cualquier función pasa

3) $\chi_0^2 >> \nu \Rightarrow \frac{\chi_0^2}{\nu} >> 1$ →
   a) Medidas incorrectas → repetir
   b) Errores muy pequeños → buscar
   c) Función modelo inadecuada → replantear teoría

Probabilidad χ^2

$$P(\chi^2 \geq \chi_0^2) = \int_{\chi_0^2}^{\infty} f(\chi^2; \nu) d\chi^2 = 1 - \int_{-\infty}^{\chi_0^2} f(\chi^2) d\chi^2$$

1) $0{,}05 \leq P(\chi^2 > \chi_0^2) \leq 0{,}95$

2) $P(\chi^2 > \chi_0^2) \geq 95\%$

3) $P(\chi_0^2) \leq 5\%$

$\ll 1\% \rightarrow$ MAL

ECUACIONES DIFERENCIALES ORDINARIAS

$$\frac{d\vec{y}}{dt} = \vec{F}(t,\vec{y}) \quad , \quad \vec{y}(t_0) = \vec{y}_0$$

Método de Euler

$$\vec{y}(t+h) \simeq \vec{y}(t) + h \cdot \vec{F}(t,\vec{y}) = y(t) + hF(t,y) \rightarrow y_{n+1} = y_n + h \underbrace{F_n}_{\frac{dy_n}{dt}}$$

Método punto medio

$$\vec{y}(t+h) \simeq \vec{y}(t-h) + 2h\vec{F}(t,\vec{y}(t)) \longrightarrow y_{n+1} = y_{n-1} + 2h\vec{F}(t_0+nh, y_n)$$

1er paso → Euler

Método predictor - corrector

$$\vec{y}(t+h) = \vec{y}(t) + \frac{1}{2}h\left[\vec{F}(t,\vec{y}(t)) + F(t+h,\vec{y}(t+h))\right]$$

$$y_{n+1} = y_n + \frac{1}{2}h\left(F(t,y_n) + F(t+h, y^*_{n+1})\right) \rightarrow$$ 1er paso y^{*o} Euler

Calif reglas integración

Euler / P. medio → extrapolar

$$y(b) = y(h) + k_1(b)h + k_2(b)h^2 + \Theta(h^3)$$

$$\hookrightarrow y(b) = \frac{4y^{(1)}(h/4) - y^{(1)}(h/2)}{3} \longrightarrow y^{(1)}(h/2) = 2y\left(\tfrac{h}{2}\right) - y(h)$$

$y(0,1)$, $y(1,0)$, $y(0,0)$

$$y^{(1)}(h/4) = 2y(h/4) - y\left(\tfrac{h}{2}\right)$$

$y(1,1)$, $y(2,0)$

$$\Rightarrow y(n,m) = \frac{2^m y(n, m-1) - y(n-1, m-1)}{2^m - 1}$$

Métodos de Runge-Kutta

Orden 2:

$$y(t+h) = y(t) + hF(t,y)(w_1+w_2) + \alpha w_2 h^2 \frac{\partial F}{\partial t} + F\beta w_2 h^2 \frac{\partial F}{\partial y} \longrightarrow \begin{cases} w_1 + w_2 = 1 \\ \alpha w_2 = 1/2 \\ \beta w_2 = 1/2 \end{cases}$$

$$= y(t) + w_1 h F(t,y) + w_2 h F(t+\alpha h, y + \beta h F(t,y))$$

$\rightarrow w_1 = w_2 = 1/2$, $\alpha = \beta = 1$ ⇒ Predictor - corrector

$\rightarrow w_1 = 0$, $w_2 = 1$, $\alpha = \beta = 1/2$ ⇒ Punto medio bis

Orden 4: → sólo 1 combin. ≥ 4

$$y(t+h) = y(t) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) + \Theta(h^5)$$

$$\begin{cases} k_1 = hF(t,y) \\ k_2 = hF\left(t+\frac{h}{2}, y + \frac{k_1}{2}\right) \\ k_3 = hF\left(t+\frac{h}{2}, y + \frac{k_2}{2}\right) \\ k_4 = hF(t+h, y+k_3) \end{cases}$$

General

$$y(t+h) = y(t) + \phi(t,y) \quad / \quad \phi(t,y) = \sum \alpha_j k_j$$

$$k_m = hF(t + a_m h, y + b_{m1}k_1 + b_{m2}k_2 + \dots + b_{m\,m-1}k_{m-1})$$

Algoritmo de Burlisch - Stoer

$$y(t+H) = \frac{1}{2}\left[\, f_j + f_{j-1} + hF(t+H, f_j)\right] \quad \boxed{h = H/j} \quad \boxed{\begin{array}{l} f_0 = y(t) = y_0 \quad f_1 \text{ Euler} \\ f_{l+1} = f_{l-1} + 2hF(t+lh, f_l) \end{array}}$$

$$\hookrightarrow y(t+h) = \frac{4^m y_{2j}^{(n)} - y_j}{4^m - 1}$$

si j pares