I. ECUACIONES DE ONDA RELATIVISTAS

- Ec. Schrödinger no relativista: $(i\partial_t + \nabla^2/2m)\psi(t,x) = 0$; $\rho_S = \psi^*\psi$; $\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0$
- Ec. de Klein-Gordon: $(\Box + m^2)\phi(x) = 0 \leftarrow \begin{cases} p_\mu = i\partial_\mu & \text{(ppo correspondencia)} \\ p_\mu p^\mu = m^2 \end{cases}$

$j^\mu = (\rho, \vec{j})$ $\quad \rho = \psi^*\psi$; $\vec{j}_{Sch} = \frac{1}{2im}(\psi^* \overleftrightarrow{\nabla}\psi)$ $\rightarrow Q = \int d^3x\, \rho$; $j^\mu_{KG} = \frac{i}{2m}(\phi^* \overleftrightarrow{\partial^\mu}\phi)$

- Ec. de Dirac ($s = \frac{1}{2}$): $(i\gamma^\mu\partial_\mu - m)\psi(x) = 0 \Rightarrow p^2 = m^2$, KG

$\times \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ // $\{\sigma^i, \sigma^j\} = 2\delta^{ij}$ // $\vec{\sigma} = \left(\begin{pmatrix}0&1\\1&0\end{pmatrix}, \begin{pmatrix}0&-i\\i&0\end{pmatrix}, \begin{pmatrix}1&0\\0&-1\end{pmatrix}\right)$

$\times\ \gamma^0_{DP} = \begin{pmatrix}\mathbb{1}_2 & 0\\ 0 & \mathbb{1}_2\end{pmatrix}$; $\gamma^i_{DP} = \begin{pmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{pmatrix}$; $\gamma'^\mu = A\gamma^\mu A^{-1}$

$\times\ \gamma^0_W = \begin{pmatrix}0 & \sigma^0\\ \sigma^0 & 0\end{pmatrix}$; $\gamma^i_W = \begin{pmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{pmatrix}$ (Realizado)

$\times\ Tr(\gamma^\mu) = 0$ $\quad \times\ Tr(\#\text{impar }\gamma) = 0$ $\quad \times\ \gamma_0^2 = \mathbb{1}$

$\times\ Tr(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}$ $\quad \times\ \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ $\quad \times\ \gamma_5^2 = \mathbb{1}$

$\times\ \{\gamma_5, \gamma^\mu\} = 0$ $\quad \times\ \gamma_i^2 = -\mathbb{1}$

$\times\ Tr(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})$ $\quad \times\ Tr(\gamma_5) = 0$

$\times\ \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

$\times\ Tr(\gamma^5\gamma^\mu) = 0$ $\quad \times\ Tr(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho) = 0$

$\times\ Tr(\gamma^5\gamma^\mu\gamma^\nu) = 0$ $\quad \times\ Tr(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = +4i\epsilon^{\mu\nu\rho\sigma}$

$\times\ Tr(\gamma^{\mu_1}\ldots\gamma^{\mu_{2m}})$ tiene $\frac{2m!}{2^m m!}$ términos $\quad \rightarrow \epsilon^{0123} = -1$

Covarianza: $L_+^\uparrow \{\Lambda^t g \Lambda = g$, $\det\Lambda = +1$, $\Lambda^0{}_0 \geq 1\}$ $\quad \Lambda$: espacio Lorentz

$x' = \Lambda x$

$\psi' = S(\Lambda)\psi \xrightarrow{\text{ec. Dirac}} S(\Lambda)^{-1}\gamma^\nu S(\Lambda) = \Lambda^\nu{}_\mu\gamma^\mu$ $\quad S$: " Dirac

$\sigma^{ij} = \begin{pmatrix}\sigma^k & 0\\ 0 & \sigma^k\end{pmatrix} = \Sigma^k \rightarrow$ spin $\frac{1}{2}$

$\overline{\psi}(x)(i\gamma^\mu\overleftarrow{\partial}_\mu + m) = 0$ $\quad \times\ \gamma^0(\gamma^\mu)^\dagger\gamma^0 = \gamma^\mu$ $\quad$ Ec. Dirac adjunta

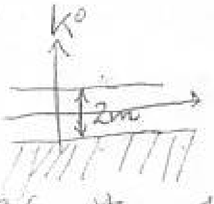
$j^\mu(x) = \overline{\psi}(x)\gamma^\mu\psi(x) \rightarrow \partial_\mu j^\mu(x) = 0$; $j^0 = \psi^\dagger\psi$

$k^0\psi(k) = (\underbrace{\gamma^0\vec{\gamma}\vec{k} + m\gamma^0}_{H})\psi(k)$ forma hamiltoniana $H = H^\dagger$

$(\gamma^\mu k_\mu - m)\psi = 0 \rightarrow \det = 0 \Rightarrow k^2 - m^2 \rightarrow \psi_2 = \frac{\vec{\sigma}\cdot\vec{k}}{E+m}\psi_1$; $\psi_1 = -\frac{\vec{\sigma}\vec{k}}{E+m}\psi_2$

Espinores $U_\pm^{1,2}$

Teoría huecos Dirac $\quad k_0 = \pm\sqrt{\vec{k}^2 + m^2}$



Conjugaci carga $\quad \psi^c = (\underbrace{i\gamma^2}_{C})\psi^*$ $\quad C(\gamma^\mu)^* C^{-1} = -\gamma^\mu$ // $\psi^c = C'\overline{\psi}^T$; $C' = i\gamma^2\gamma^0$

Propiedades de espinores elementales:

$\bar{u}^r u^{r'} = 2m\delta^{rr'}$; $\bar{u} v = 0$; $u^{+r} u^{r'} = v^{r+} v^{r'} = 2E\delta^{rr'}$

$\bar{v}^r v^{r'} = -2m\delta^{rr'}$; $\bar{v} u = 0$;

$\Lambda_\pm = \dfrac{\pm\gamma^\mu k_\mu + m}{2m}$; $\Lambda_\pm^2 = \Lambda_\pm$; $\Lambda_+ + \Lambda_- = \mathbb{1}_4$

$\sum_r u^r \bar{u}^r(k) = \not{k} + m$; $\sum_{r'} v_r \bar{v}^r = \not{k} - m$

$\psi(x) = \sum_k (a_r(k) u^r(k) e^{-ikx} + b_r^+(k) v^r(k) e^{ikx})$; $\sum_k = \dfrac{1}{(2\pi)^3}\int \dfrac{d^3k}{2|k^0|} = \int d^4k\, \delta(k^2 - m^2)$

$L^{\uparrow \ldots \Lambda^0}_{\pm \ldots \det}$ $\to$ Vierergruppe e, T, P, PT

Parid $(1, -1, -1, -1)$ $\to$ $\psi'(x') = S(P)\psi(x)$
$\psi'(x) = S(P)\,\psi(x^0, -\vec{x})$

$S^{-1}(P)\gamma^0 S(P) = \gamma^0$
$S^{-1}(P)\gamma^i S(P) = -\gamma^i$ $\to S = \eta_P \gamma^0$, $\eta_P = \pm 1, \pm i$

Bilineales de Dirac

$\bar{\psi}\Gamma\psi(x)$ $\to$ alude a cómo se transforme bajo P

$(\bar{\psi}\Gamma\psi)'(x) = \text{sig}\,(\bar{\psi}\Gamma\psi)(Px)$

$\boxed{\text{sig} = -1 \to \text{pseudo}}$

k_{eff} $\mu = 0 \to \text{sig} = 1$
$\mu = i \to \text{sig} = -1$

$\Gamma = \mathbb{1}$	Escalar	1
γ^5	Pseudoescalar	1
γ^μ	Vectorial	4
$\gamma^\mu\gamma^5$	Pseudovectorial	4
$\sigma^{\mu\nu}$	Tensorial	6

Límite NR:

$E_c \psi_e = \dfrac{\vec{p}^2}{2m}\psi_e$

Átomo hidrógeno $E(n, j)$ ... espín total

Ecs. de Weyl $(m=0)$

$\gamma^\mu_W = \begin{pmatrix} 0 & \sigma^\mu \\ \tilde{\sigma}^\mu & 0 \end{pmatrix}$ $\quad \sigma^\mu = (\mathbb{1}_2, \sigma^i)$
$\tilde{\sigma}^\mu = (\mathbb{1}_2, -\sigma^i)$

$\underbrace{\dfrac{\vec{\sigma}\cdot\vec{k}}{|\vec{k}|}}_{\text{helicidad}} \psi^R_L = \pm \psi^R_L$ ($k^0 > 0$) $\to$ $m = 0 \to$ NO ó $+1$ ó -1
no los intermedios

$\infty \; \frac{1}{2}(1 \pm \gamma_5)$

II. FORMALISMO CANÓNICO DE CUANTIZACIÓN

q_i, p_j $\quad i=1,\dots,n$ (formula hamiltoniana)

$p_j = \frac{\partial L}{\partial \dot q_j}$ $\quad \to H = \dot q_i p_i - L$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_i}\right) - \frac{\partial L}{\partial q_i} = 0$ $\quad L \boxed{\frac{\partial H}{\partial p_i} = \dot q_i \; ; \; \frac{\partial H}{\partial q_i} = \dot p_i}$ Ecs de Hamilton

$$\frac{dF(q,p,t)}{dt} = \frac{\partial F}{\partial t} + \{F,H\} = \frac{\partial F}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial F}{\partial p_i}\frac{\partial H}{\partial q_i}$$

$\{q^i, p_j\} = \delta^i_j \; ; \; \{q^i, q^j\} = 0 = \{p^i, p_j\}$

MQ: $\frac{dO_H}{dt} = \frac{\partial O_H}{\partial t} + \frac{1}{i\hbar}[O_H, H]$

$\boxed{1^a C}$ Asociar $\{\ ,\ \} \to \frac{1}{i\hbar}[\ ,\ ]$ paréntesis de Dirac (variables, obs.) 1^a

$[q_i, p_j] = i\hbar\delta^i_j$

$\boxed{2^a C}$ (campos) 2^a

$\mathcal{L}(\varphi_i, \partial_\mu\varphi_i) \to \mathcal{H} = \pi_i \cdot \dot\varphi_i - \mathcal{L}$

$\pi_i = \frac{\partial\mathcal{L}}{\partial\dot\varphi_i}$

si φ bosónica:

$[\varphi(x^0,\vec x), \pi(x^0,\vec y)] = i\hbar\,\delta(\vec x - \vec y)$ $\quad ; \quad \hat\pi(x^0,\vec y) = -i\hbar\frac{\delta}{\delta\varphi(x^0,\vec y)}$

φ fermiónica $\quad \to$ por microcausalidad

$\{\psi(x^0,\vec x), \psi(x^0,\vec y)\} = i\hbar\,\delta(\vec x - \vec y)$ $\quad [O(x), O(y)] = 0 \quad si\ (x-y)^2 < 0$

observables (bilineales Dirac)

$I = \int d^4x\, \mathcal{L}(\varphi, \partial_\mu\varphi)$

$$\partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)} - \frac{\partial\mathcal{L}}{\partial\varphi} = 0$$

Tma. Noether

$\delta I = 0$ para una cierta simetría $\to \exists\, j^\mu \,/\, \partial_\mu j^\mu = 0$ (conserva corriente) para φ que satisface E-L

$\delta\mathcal{L} = \partial_\mu F^\mu \,/\, \delta I = 0$ (sólo contribuye a Noether) $\quad$ + Carga conser. $Q = \int d^3x\, j^0$

$\delta\phi = -iq\alpha\phi \quad \to$ buscar $\partial_\mu F^\mu \to$ fase, transf. U(1)

$\to j^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\cdot\frac{d}{ds}\phi_i \quad \left(\frac{d\mathcal{L}}{ds} = 0\right)$ $\quad SU(n) = \{A_{n\times n} \,|\, A^\dagger A = \mathbb{1}_n, \det A = 1\}$

$j^\mu_{KG} = i\phi^*\overleftrightarrow{\partial_\mu}\phi \quad (\phi_{KG}) \quad ; \quad j^\mu_D = \bar\psi\gamma^\mu\psi \quad (\psi_D)$

$= -iq\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\varphi - \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi^*)}\varphi^*\right)$

$j^\mu = \left[-g^{\mu\nu}\mathcal{L} + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\frac{\partial\varphi}{\partial x_\nu}\right]\delta x_\nu + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\delta\varphi \quad \to$ transf. Poincaré $(\epsilon^\mu, \Lambda^\mu{}_\nu)$

Traslac. $\to \Theta^{\mu\nu}$ tensor E-impulso ; $\Theta^{00} = \mathcal{H}$

Tr. Lorentz $\to M^{\mu}{}^{\nu\rho}$

III. CUANTIZACIÓN DE CAMPOS LIBRES

• $\boxed{\text{Klein-Gordon}}$: $(\Box + m^2)\phi(x) = 0$; $\mathcal{L} = (\partial_\mu \phi^*)(\partial^\mu \phi) - m^2\phi^*\phi$

$\pi(x) = \dot\phi^* = \frac{\partial \mathcal{L}}{\partial(\partial_0\phi)}$; $j^0 = i\underset{\text{↳ dimensiones de masa}}{q}(\pi^*\phi^* - \pi\phi) \to Q = \int d^3x\, j^0$

$[\phi(x), \pi(x')] = i\delta(\vec x - \vec x')$

$[\phi(x), \dot\phi^*(x')]_{x^0=x'^0} = i\delta(\vec x - \vec x')$

$[Q, \phi(x)] = -q\phi$; $[Q,\phi^*] = q\phi^*$; $[a, a^\dagger] = \Delta_{\vec k\vec k'}$; $[a,a] = 0 = [a^\dagger,a^\dagger]$

$Q(\phi|\xi\rangle) = (N-1)q(\phi|\xi\rangle) \to \phi$ - destruye una unidad de carga
ϕ^* - crea

$\phi(x) = \sum_k (a(k)\underbrace{e^{-ikx}}_{f_k} + b^\dagger(k)\underbrace{e^{ikx}}_{f_k^*}) = \phi^{(+)} + \phi^{(-)}$; $\sum_k = \frac{1}{(2\pi)^3}\int \frac{d^3k}{2k^0}$

prod. escalar
$(f_k(x), f_{k'}(x)) = i\int d^3x\, f_k^* \overleftrightarrow{\partial_0} f_{k'}(x) = (2\pi)^3\delta^{(3)}(\vec k - \vec k')\cdot 2E = \Delta_{\vec k\vec k'}$

$(f_k^*(x), f_{k'}^*(x)) = -\Delta_{\vec k\vec k'}$; $(f, f^*) = 0$ → elementos base, a, b → proyección

↳ $a(\vec k) = (f_k(x), \phi(x))$; $b^\dagger(\vec k) = (-f_k^*(x), \phi(x))$

$\phi(x) = -\int_{x_0} d^3x' \Delta(x-x')\overleftrightarrow{\partial_0}\phi(x') \Rightarrow \Delta(x-x') = -i\sum_k \left(e^{-ik(x-x')} - e^{ik(x-x')}\right) = \Delta^+(x-x') + \Delta^-(x-x')$

↳ problema de Cauchy: $\left(\oint_C dx \frac{F(x)}{x-a} = 2\pi i F(a)\right)$

$\Delta(x) = -\frac{1}{(2\pi)^4}\int d^4k \frac{e^{-ikx}}{k^2-m^2}$

$[\phi(x), \phi^*(y)] = i\Delta(x-y) = 0$ si $(x-y)^2<0 \Rightarrow$ Microcausalidad

$[(f,\phi),\underbrace{(g,\phi)^*}_{(\phi,g)}] = (f,g)$; $[(f,\phi),(g,\phi)] = 0$

$[a(\vec k), a^\dagger(\vec k')] = [b(\vec k), b^\dagger(\vec k')] = \Delta_{\vec k\vec k'}$; resto $= 0$

Espacio de Fock para K-G

$Q(a^\dagger(\vec k)|0\rangle) = q(a^\dagger(\vec k)|0\rangle)$

a, b destruye $\begin{cases} p \\ ap \end{cases}$ de masa m, momento k^μ y carga $\begin{cases} q \\ -q \end{cases}$; $a^\dagger(\vec k)|0\rangle = |k\rangle$

a†, b† crea $\begin{cases} p \\ ap \end{cases}$ $\begin{cases} q \\ -q \end{cases}$; $b^\dagger(\vec k)|0\rangle = |\underline{k}\rangle$

$[Q, \underset{a}{a^\dagger(\vec k)}] = \underline{q}\, a^\dagger(\vec k)$; $[Q, \underset{b}{b^\dagger(\vec k)}] = -\underset{+}{q}\, b^\dagger(\vec k)$ → ϕ destruye con a, b†
ϕ^* crea con a†, b

$[P^\mu, \phi] = -i\partial^\mu\phi$; $[P^\mu, a^\dagger(\vec k)] = k^\mu a^\dagger(\vec k)$; $[P^\mu, b(\vec k)] = -k^\mu b(\vec k)$; $p^\mu p_\mu = m^2$, k^0

$[P^\mu, \phi^*] = -i\partial^\mu\phi^*$; $[P^\mu, a(\vec k)] = -k^\mu a(\vec k)$; $[P^\mu, b^\dagger(\vec k)] = k^\mu b^\dagger(\vec k)$

Densidad de nº de partículas y antipartículas

$N = a^\dagger a(\vec k)$; $\bar N = b^\dagger b(\vec k)$; $[AB, C] = A[B,C] + [A,C]B$

$[N_k, a^\dagger(\vec k')] = a^\dagger(\vec k)\Delta_{\vec k\vec k'}$; $[\bar N_k, b^\dagger(\vec k)] = b^\dagger(\vec k)\Delta_{\vec k\vec k'}$; $[N_k, (\underset{b^\dagger}{a^\dagger}(k'))^n] = n(\underset{b^\dagger}{a^\dagger}(k'))^n\Delta_{kk'}$

$N(a^\dagger(k)^n|0\rangle) = n\, a^\dagger(k)^n|0\rangle$

$|k_1\ldots k_n\rangle = \frac{1}{\sqrt{n!}} a^\dagger(k_1)\ldots a^\dagger(k_n)|0\rangle \to$ Estado de n partículas

Proyección ; $\langle k|k'\rangle = \Delta_{\vec k\vec k'}$; $\langle k_1', k_2'|k_1k_2\rangle = \Delta_{k_1'k_1}\Delta_{k_2'k_2} + \Delta_{k_1'k_2}\Delta_{k_1k_2'}$

$Q = q\sum_k (a^\dagger a - b b^\dagger) \to :Q: = q\sum_k(a^\dagger a - b^\dagger b) = q(N - \bar N)$

Estadística

$:P^\mu: = \sum_k k^\mu(a^\dagger a + b^\dagger b) > 0$

$\langle x|k\rangle = e^{-ikx}$: f. ondas ; $\phi^{(+)*}(x)|0\rangle = |k\rangle$

• $\langle x_1x_2|k_1k_2\rangle$ → f. ondas simétrica → bosones ; Dirac: AS (fermiones)

• Cuantizamos al revés → viola microcausalidad ⇒ ∃ una conexión espín-estadística

$\boxed{\text{Dirac}}$

$\mathcal{L} = -\bar\psi(-i\gamma^\mu \overrightarrow{\partial_\mu} + m)\psi \xrightarrow{E-L}$ ec. Dirac

$\pi = \frac{\partial \mathcal{L}}{\partial \dot\psi} = i\bar\psi\gamma^0 = i\psi^+$

$\{\psi_\alpha(x), \bar\psi_\beta(x')\}_{x^0=x^{0'}} = \gamma^0_{\alpha\beta}\,\delta(\vec{x}-\vec{x}')$ $\quad \{\psi, \psi^+\} = \delta_{\alpha\beta}\,\delta(\ )$

$\Theta^{\mu\nu} = i\bar\psi\gamma^\mu\partial^\nu\psi \to P^\mu = \int d^3x\, \psi^+\partial^\mu\psi$; $\underset{\vec{M},\vec{N}}{H} = P^0 = \int d^3x\, \bar\psi\,(-i\vec\gamma\vec\nabla + m)\psi$

$Q = q\int d^3x\, \psi^+\psi \leftrightarrow j^\mu = q\bar\psi\gamma^\mu\psi$

$\psi(x) = \sum_{\vec{k},r}(a_r(\vec{k})\underbrace{u^r(\vec{k})e^{-ikx}}_{f_{\vec{k}}^r(x)} + b_r^+\underbrace{v^r(k)\cdot e^{ikx}}_{g_{\vec{k}}^r(x)}) = \psi^{(+)} + \psi^{(-)}$ $\quad (f_k^r(x), f_{k'}^s(x)) = (g_k^r(x), g_{k'}^s(x)) = \delta^{rs}\Delta_{kk'}$

$(f, g) = 0$

↳ pr. escalar Dirac $\neq$ K-G $\quad \int d^3x\, f^+ f \quad \to q\, j^\mu \neq$

$a_r(\vec{k}) = (f_{k,r}(x), \psi(x))$; $b_r^+(\vec{k}) = (g_{k,r}(x), \psi(x))$

$\psi(x) = -i\int d^3x'\, S(x-x')\gamma^0\psi(x')$

$-iS(x-x') = \sum_k (f_k^r(x)\bar f_{k,r}(x') + g_k^r(x)\bar g_{k,r}(x')) = i(i\gamma^\mu\partial_\mu + m)\Delta(x-x')$

S satisface ec. Dirac

$H = \sum_k E(a_r^+a_r + b^+b)$; $\dot\psi = i[H, \psi(x)]$ ec. Heisenberg

$[AB, C] = A\{B,C\} - \{A,C\}B$

$[N_{k'}, a_k] = -a\,\Delta_{kk'}$; $[b^+b_{k'}, b^+_{k'}] = b^+\Delta_{kk'}$

$\{\psi_\alpha(x), \psi_\beta^+(y)\}_{x_0=y_0} = iS_{\alpha\beta}(x-y)$

$\{(g,\psi), (g,\psi)^+\} = (g,g)$; $((f,\psi),(g,\psi)) = 0$

$\{a_r(\vec{k}), a_s^+(\vec{k}')\} = \delta_{rs}\Delta_{\vec{k}\vec{k}'}$

$\to$ resto $= 0$

$\{b_r(\vec{k}), b_s^+(\vec{k}')\} = \delta_{rs}\Delta_{\vec{k}\vec{k}'}$

$:Q: = q(N - \bar N)$; $:P^\nu: \sum_k k^\nu(a^+a + b^+b)$

$[Q, \psi] = -q\psi$; $[P^\nu, \psi] = -i\partial^\nu\psi$; $[P^\nu, a_r] = -k^\nu a_r$; $[P^\nu, b_r] = -k^\nu b_r$

$[Q, \psi^+] = +q\psi^+$ $[P^\nu, \psi^+] = -i\partial^\nu\psi^+$; $[P^\nu, a_r^+] = k^\nu a_r^+$; $[P^\nu, b_r^+] = k^\nu b_r^+$

$[Q, a_r^+] = q\,a_r^+$; $[Q, b^+] = -q\,b^+$; $[Q, a_r] = -q\,a_r$; $[Q, b] = q\,b$

$a_r^+(k)|0\rangle = |\vec{k}, r\rangle$

$\langle 0|\phi^{(+)}(x) = \langle x| \quad\to\quad \langle x|k,r\rangle = f_{k,r}(x)$

$\sum_s a_s(\vec{k}')\,f_{k'}^s(x')\,a_r^+(k) = f_{k,r}$

$\langle x_1 x_2|k_1 r_1 k_2 r_2\rangle \Rightarrow AS$

Tma. Conexión Espín-Estadística

$O(x) = \psi_a(x)\Gamma^{ab}\psi_b(x) \to$ bilineal

Cond. de microcausalidad: $[O(x), O(y)] = 0$ para $(x-y)^2 < 0$

Bosones:

$[\psi_a(x), \psi_b(x')] = 0$

Fermiones:

$\{\psi_a(x), \psi_b(x')\} = 0$ $\quad$ para $(x-y)^2 < 0$

$\to$ Consecuencia de que la teoría es relativista | y t.ª local de campos

Spin entero $\quad$ Spin semientero

$[\ ,\ ]_{poiss} \to \frac{1}{i\hbar}[\ ,\ ]$

$[AB, CD] = -AC[D,B] - A[C,B]D - C[D,A]B - [C,A]DB$

$= -AC\{D,B\} + A\{C,B\}D - C\{D,A\}B + \{C,A\}DB$

- $\boxed{\text{Campos de Proca}}$ $V(m, S=1)$ y $\boxed{\text{Maxwell}}$ $\Delta(m=0, h=1)$

Proca complejo con masa:

$$\mathcal{L}(x) = -\frac{1}{2} F_{\mu\nu}^{*} F^{\mu\nu} + m^2 \varphi_\nu^* \varphi^\nu \rightarrow F_{\mu\nu} = \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu$$

$$\rightarrow \partial_\mu F^{\mu\nu} + m^2 \varphi^\nu = 0$$

Condición subsidiaria $\partial_\nu \varphi^\nu = 0$ $(4-1=3 \text{ dof})$ $\rightarrow k_\nu \varphi^\nu(k) = 0$ $\rightarrow (m \neq 0)$!!

Stma en reposo $\rightarrow k^\mu = (m, \vec{0})$ $\Rightarrow \varphi^0 = 0$

$$\mathcal{L}_{proca} = -(\partial_\mu \varphi_\nu^*)(\partial^\mu \varphi^\nu) + m^2 \varphi_\nu^* \varphi^\nu$$

$\Theta^{\mu\nu}$, H, ... $j^\mu = -iq\,(\varphi^{*\lambda} \overleftrightarrow{\partial^\mu} \varphi_\lambda)$; $m^{\mu\nu\lambda}$, $(\Sigma_{\mu\nu})_{\rho\sigma}$, $S^{\nu\lambda}$

$$\varphi_\mu(x) = \sum_{\vec{k}, \lambda=1,2,3} \Big(\underbrace{\epsilon_\mu{}^\lambda(\vec{k})\, a_\lambda(\vec{k})\, e^{-ikx}}_{f_k^{\mu\lambda}} + \underbrace{\epsilon_\mu{}^\lambda(k)^*\, b^\dagger{}_\lambda(\vec{k})\, e^{ikx}}_{f_k^{\mu\lambda *}} \Big)$$

$k^\mu \epsilon_\mu{}^\lambda(\vec{k}) = 0 \quad \forall \lambda$; $\epsilon_\mu \epsilon^{\mu\nu} = -1$; $\epsilon_\mu{}^\lambda \epsilon^{*\mu\lambda'} = -\delta^{\lambda\lambda'}$; $\vec{n}^\lambda(\vec{k}) \vec{n}^{\lambda'}(\vec{k}) = \delta^{\lambda\lambda'}$

$$\epsilon_\mu{}^\lambda(\vec{k}) = \left(\frac{\vec{n}^\lambda \vec{k}}{m}, \vec{n}^\lambda + \frac{\vec{k} \cdot (\vec{k} \cdot \vec{n}^\lambda)}{m(E+m)} \right)$$

$\Lambda_{\mu\nu} = \sum_\lambda \epsilon_\mu{}^\lambda \epsilon_\nu{}^\lambda(k)$; $\Lambda_{\mu\nu} \epsilon^{\nu\lambda'} = \cancel{\sum_\lambda \epsilon_\mu^\lambda(k) \epsilon_\nu^\lambda(k)\epsilon^{\nu\lambda'}} - \epsilon_\mu{}^{\lambda'}(k)$; $k^\mu \Lambda_{\mu\nu} = 0$

$$\parallel$$

$$-\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{m^2} \right)$$

$a^\lambda(k) = (f_k^{\lambda\nu}(x), \varphi_\nu(x))$; $b^{\dagger\lambda}(\vec{k}) = -(f_k^{\lambda\nu}(x)^*, \varphi_\nu(x))$

$$P_{\mu\nu}(x-x') = -\left(g_{\mu\nu} + \frac{\partial_\mu \partial_\nu}{m^2} \right) \Delta(x-x')$$

$[\varphi_\mu(x), \pi^\nu(y)]_{x^0=y^0} = i\, g_\mu^\nu\, \delta(\vec{x}-\vec{x}')$; $[\varphi_\mu(x), \dot{\varphi}_\nu^*(y)]_{x^0=y^0} = -i g_{\mu\nu} \delta(\vec{x}-\vec{x}')$

La rel. conmutac. a t. $\neq$ s

$$[\varphi_\mu(x), \varphi^*_\nu(y)]_{x^0 \neq y^0} = i\, P_{\mu\nu}(x-y)$$

$[a^\lambda(\vec{k}), a^{\lambda'}(\vec{k}')^\dagger] = [b^\lambda(\vec{k}), b^{\dagger\lambda'}(\vec{k}')] = \Delta_{\vec{k}\vec{k}'} \delta^{\lambda\lambda'}$; resto $= 0$

$P^\nu = \sum_{k,\lambda} k^\nu (a^{\dagger\lambda} a^\lambda + b^{\dagger\lambda} b^\lambda)$; $Q = q(N - \bar{N})$; S_z ; $\rightarrow (\Box + m^2)\Delta = 0$

Maxwell: $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$; $A^\mu = (\phi, \vec{A})$ $\rightarrow \partial_\mu F^{\mu\nu} = j^\nu$ Ecs. de M. homog.

Tr. Gauge: $A'_\mu = A_\mu + \partial_\mu \lambda$; $F'^{\mu\nu} = F^{\mu\nu}$

- Lorentz $\partial_\mu A^\mu = 0$, 2ª condición $\Box \lambda = 0$ $\rightarrow$ 2 d.o.f. $\rightarrow$ helicid. ± 1 ; $\Box A_\mu = j_\mu$

$A_\mu = (0, A_1, A_2, 0)$

- Coulomb: $\vec{\nabla} \cdot \vec{A} = 0$ $\rightarrow$ no covariante

$\mathcal{L}_{free} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ $\rightarrow$ $\pi^\mu = F^{\mu 0}$; $\pi^0 = 0$

$-\frac{\lambda}{2}(\partial_\mu A^\mu)^2$ ó $\mathcal{L}_F = -\frac{1}{2}(\partial_\mu A_\nu)(\partial^\mu A^\nu)$; $\partial^\mu A_\mu = 0$; $\Box A^\mu(x) = 0$

$$A_\mu(x) = A_\mu^*(x) = \sum_{\vec{k}, \lambda} \Big(a^\lambda(\vec{k}) \underbrace{f_{\mu \vec{k}}^\lambda(x)}_{\epsilon_\mu{}^\lambda(k) e^{-ikx}} + a^{\lambda\dagger}(\vec{k}) \underbrace{f_{\mu \vec{k}}^{\lambda *}(x)}_{\epsilon_\mu{}^{\lambda *}(k) e^{ikx}} \Big)$$

$\epsilon_\mu{}^\lambda \epsilon^{*\mu\lambda'} = g^{\lambda\lambda'}$; $\epsilon_\mu{}^\lambda \epsilon_\nu{}^\lambda = g_{\mu\nu}$

$(f_\mu, g_\mu) \overset{\text{def. prod. escalar}}{=} -i \int d^3x\, f_\mu^*(x) \overleftrightarrow{\partial_0} g^\mu(x) = (g, f)^*$

$(f, f') = -g^{\lambda\lambda'} \Delta_{\vec{k}\vec{k}'} = -(f^*, f'^*)$; $(f, f'^*) = (g, g^*) = 0$; $a^\lambda(\vec{k}) = (-g^{\lambda\lambda})(f_{\mu k}{}^\lambda, A_\mu)$

$a^{\lambda\dagger}(\vec{k}) = (-g^{\lambda\lambda})(f_{\mu k}^*, A_\mu)$

$$D(x-x') = -\frac{i}{\ldots} \sum_{\vec{k} \neq 0} \left(e^{-ik(x-x')} - e^{ik(x-x')} \right)$$

$[A_\mu(x), A_\nu(y)] = -i g_{\mu\nu} D(x-y)$; a t = s: $-i g_{\mu\nu} \delta(\vec{x}-\vec{x}')$

$[a, a'^\dagger] = -g^{\lambda\lambda'} \delta_{kk'}$; $[a, a] = 0 = [a^\dagger, a^\dagger]$

Gupta-Bleuler $\rightarrow$ $\partial_\sigma A^{\sigma(+)} |\psi\rangle = 0$ (parte destrucción) $\rightarrow$ $\langle \psi | \partial^\sigma A_\sigma | \psi \rangle = 0$

$$H = \sum_{k,\lambda} -g^{\lambda\lambda'} E\, a^\dagger a$$

fotones temporales, transversos, longitudinales...

T. IV - INTERACCIONES

$\mathcal{L}_{free} \xrightarrow{\text{Ppo inv. Gauge local}} \mathcal{L}_{int}$

derivada covariante; conexión

$(D_\mu\psi)' = e^{-iq\chi(x)} D_\mu \psi \quad ; \quad D_\mu = (\partial_\mu + iqA_\mu) \quad ; \quad A'_\mu = A_\mu + \partial_\mu \chi(x)$

- KG int A_μ : $\mathcal{L}_{int} = -\frac{1}{2}(\partial_\mu A_\nu)(\partial^\mu A^\nu) \rightarrow$ ec. E-L, π^ν, ... $[Q, \;]$

$\mathcal{L}_I = -iq\,[\phi^* \overleftrightarrow{\partial_\mu} \phi] A^\mu + q^2 A_\mu A^\mu \phi^*\phi \quad ; \quad (\Box + m^2)\phi(x) = j(x)$

- Dirac : $\mathcal{L}_I = -q(\overline{\psi}\gamma^\mu\psi)A_\mu = -j^\mu A_\mu$

$[P_\mu, A_\nu] = -i\partial_\mu A_\nu, \quad [P_\mu, \psi] = -i\partial_\mu\psi \quad ; \quad (-i\gamma^\mu\partial_\mu + m)\psi(x) = j(x)$

Matriz $S = \sum_n |n\rangle_{in} \, {}_{out}\langle n| \quad ; \quad S^+S = 1 \quad ; \quad S|n_{out}\rangle = |n_{in}\rangle \; ; \; S^+|n_{in}\rangle = |n_{out}\rangle$

$[S, P_\mu] = 0 \rightarrow$ conservac. del mom, carga

$\psi_{in} = U(t)\psi(x)U^{-1}(t)$

$\hookrightarrow i\dot{U}U = H_I + E_0 \rightarrow \dot{U} = -iH_I U(t) \quad ; \quad U(t) = \sum_{n=0}^{\infty} U_n(t) \rightarrow U_{n+1} = -iH_I U_n$

$U(t) = 1 + \sum_{n=1}^{\infty}(-i)^n \int_{-\infty}^{t}\int_{-\infty}^{t_1} \dots \int_{-\infty}^{t_{n-1}} H_I(t_1) \dots H(t_n) = 1 + \sum I_n$

$I_n = \int_{t_0}^{t}\int_{t_0}^{t} \dots \Theta(\;)$ — producto cronológico $P(H_I \dots) = \sum_{perm} \Theta \dots H$

$S = P\exp\left\{-i\int_{-\infty}^{+\infty} dt\, H_I(t)\right\} = P\left(+i\int d^4x\, \mathcal{L}_I(x)\right)$

Producto T-ordenado : N-ordenado

$T(\phi(x), \phi^*(y)) = \theta(x_0 - y_0)\phi(x)\phi^*(y) \pm \theta(y^0 - x^0)\phi^*(y)\phi(x)$

$N(\phi(x)\phi^*(y)) = \phi(x)\phi^*(y) - [\phi^+(x), \phi^{*-}(y)]$

$T(\phi(x)\phi^*(y)) = N(\phi(x)\phi^*(y)) + \overbrace{\phi(x), \phi^*(y)} = \langle 0|\overbrace{\phi(x)\phi^*(y)}|0\rangle$

$\langle 0|T|\phi(x)\phi^*(y)|0\rangle = i\Delta_F(x-y) \quad \parallel \quad$ Dirac $iS_F(x-y)$

observable : bilin Dirac $\rightarrow$ puedes cambiar $P \leftrightarrow T \Rightarrow \boxed{S = T\exp i\int d^4x\, \mathcal{L}_I(x)}$

$\overbrace{\phi(x)\psi(y)} = 0, \dots = 0$

$\Delta_F(x-y) = \frac{1}{(2\pi)^4}\int_{C_F} d^4k \frac{e^{-ik(x-y)}}{k^2 - m^2} \quad ; \quad S_F(x) = \left(\frac{1}{2\pi}\right)^4 \int d^4k \frac{\not{k} + m}{k^2 - m^2 + i\epsilon}$

Tma. de Wick

$T(\varphi_1(x_1)\varphi_2(x_2)\dots\varphi_n(x_n)) = N(\varphi_1(x_1)\dots\varphi_n(x_n)) + N(\overbrace{\varphi_1\varphi_2}\dots)$ — todas contr. posibles de 2 campos

$+ N(\sqcap\sqcap) + \dots \quad \frac{n}{2}$ ó $\frac{n-1}{2}$ veces contraídos

$N[\varphi_1\overbrace{\varphi_2\varphi\dots}] = \delta_P \overbrace{\varphi_1\varphi_2} N[\;] \quad \delta_P = \pm 1$ según $(-1)^{\text{cruces}}$

+ OMITIR contracciones a tiempos = de $\overline{\psi}$ con ψ

$S = \sum S_n \; ; \; S_n = \frac{i^n}{n!}\int d^4x_1 \dots d^4x_n T(\mathcal{L}_I(x_1)\dots\mathcal{L}_I(x_n))$

si N ordenado $\langle 0|N|0\rangle = 0$

$\left(\frac{i}{(2\pi)^4}\right)^{m-n}$ m: líneas internas; n: nº de vértices $\longrightarrow$ prefactor

$m - (n-1) = \ell \;\hookrightarrow \left(\frac{i}{(2\pi)^4}\right)^{\ell - 1}$

$m - n = 1$:

$\langle S_c\rangle = -i(2\pi)^4\delta^{(4)}(\text{Conservac}) \cdot \langle A_c\rangle$

Reglas de Feynman <Ac>:

bosones:

Línea externa → 1

" interna → $\dfrac{1}{q^2-m^2+i\epsilon}$

Vértice • g

Lazo ○ $\left(\frac{i}{2\pi}\right)^4 \int dl$

Factor simetría $\dfrac{1}{n_s}$

Factor (−) x cruce lín. ferm. y cada lazo

Fermiones: línea ferm. entrante-saliente u_r $\bar{u}_r$

línea antif. entrante-saliente $\bar{v}_r$ v_r

L. fe. interna: $\dfrac{\not{q}+m}{q^2-m^2+i\epsilon}$

L. fotónica entr/sal: $\epsilon_\mu / \epsilon_\mu^*$

L. " interna: $\dfrac{-g_{\mu\nu}}{q^2+i\epsilon}$

A veces: A_μ → campo externo clásico, Coulomb, sacas fuera

$$P_{i\to f} = \frac{|\langle f|S|i\rangle|^2}{\langle i|i\rangle} \sim vT|\;|^2 \dashrightarrow \sigma T = \frac{P_{i\to f}/T}{\text{Flujo}} = \frac{P_{i\to f}/T}{v\cdot ST}$$

$$\vec{P}_{CMS} = \frac{1}{2\sqrt{s}}\sqrt{\lambda(s,m_1^2,m_2^2)}$$

suma espines finales, promedio iniciales

$$\sigma_T^{i\to f} = \frac{1}{2\sqrt{\lambda}} \sum_f \int \frac{1}{(2\pi)^{3n'-4}} \prod_{j=1}^{n'} \delta^+(p_j^2-m_j^2)\cdot\delta\left(\sum p_i - p_f\right) \overline{\sum_s} |\langle f|T|i\rangle|^2$$

$$\Gamma = \frac{1}{2p_N}\sum\int \quad ''$$

$$1+2\to 3+4:\ \sigma = \frac{1}{2\sqrt{\lambda}}\frac{1}{(2\pi)^2}\int \frac{d^3p_3\, d^3p_4}{4E_3E_4}\delta^{(4)}\cdot\overline{\sum_s}|\;|^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{CMS} = \frac{1}{64\pi^2 s}\sqrt{\frac{\lambda(s,m_3^2,m_4^2)}{\lambda(s,m_1^2,m_2^2)}}\;\overline{\sum}|T|^2$$

$$1\to 3+4:\quad \Gamma = \frac{\sqrt{\lambda(s,m_3^2,m_4^2)}}{64\pi^2 m_1^3}\int d\Omega\,\overline{\sum_s}|T|^2$$

$\gamma^\mu c \gamma_\mu = 4c$

$\gamma^\mu \not{a} \gamma_\mu = -2\not{a}$

$\gamma^\mu \not{a}\not{b}\gamma_\mu = 4ab$

$\gamma^\mu \not{a}\not{b}\not{c}\gamma_\mu = -2\not{c}\not{b}\not{a}$

T. VI - RENORMALIZACIÓN EN QED

- teorías Gauge de Yang-Mills ; ppo Gauge $\leftrightarrow$ ppo simetría dinámico (de la interacc)

- DIVERGENCIAS EN QED (integrales que no convergen y $\frac{1}{0}$)
  - Diagramas de vacío (scatt. Compton, 2º orden)
  - Diverges infrarrojas (k bajo $\rightarrow$ no mides, CUT), soft photon
  - " ultravioletas (k grande) $\rightarrow$ Renormalizac

- Grado superficial de divergencia de un diagrama (cuenta de potes de Dyson)

$$D = -4(n-1) + 3F_i + 2B_i$$

n: nº de vértices
$F_{i/e}$ líneas ferm. internas / externas
$B_{i/e}$ " bosón.

Para QED:

$$D = 4 - \frac{3}{2} F_e - B_e$$

$< 0 \rightarrow$ converge ∅ No diverge primitiva∅

La Diagrama primitiva∅ divergente: si se corta cualquier línea interna: ya es convergente

- AutoE e^-, γ  + órdenes superiores
- Diagrama Δ

× $D = 4$ ($F_e = B_e = 0$) $\rightarrow$ d. vacío (ignorar)
× $D = 3$ ($F_e = 0, B_e = 1$) $\rightarrow$ 0 (tma. Fury)
× $D = 2$ ($F_e = 0, B_e = 2$) $\rightarrow$ AutoE γ $\rightarrow D_{eff} = 0$ (logarítmica)
× $D = 1$ ($F_e = 0, B_e = 3$) $\rightarrow$ 0 (tma. Fury)
($F_e = 2, B_e = 0$) $\rightarrow$ AutoE e^- " "
× $D = 0$ ($F_e = 0, B_e = 4$) $\rightarrow$ Dispersión luz-luz

---

$$\frac{1}{x} - \frac{1}{x} y \frac{1}{x} + \dots = \frac{1}{x+y}$$  + ...

$$m = m_0 + \delta m \quad ; \quad Z_2 = \frac{1}{1 - e_0^2 B} \quad ; \quad e = e_0 Z_3^{1/2}$$

$$\psi' = Z_2^{1/2} \psi$$

Identif de Ward - Takahashi / Slavnov - Taylor

$$\Lambda_\mu(p, p) = \frac{\partial}{\partial p^\mu} \Sigma W_2, p \quad ; \quad Z_1 = Z_2$$

Tª Fermi Int. débil

$$D = 4 + 2n - \frac{3}{2} N_e - \frac{3}{2} L_e$$

$\rightarrow$ depende orden n y

Mto magnético anómalo

T. ? - TRANSFORMACIONES C, P, T

$\boxed{\mathcal{P}}$: $P(x^0, \vec{x}) \to (x^0, -\vec{x})$ Paridad

$\vec{k}, \vec{v} \to -\vec{v}, \vec{k}$

$\vec{L} = \vec{r} \times \vec{p} \to \vec{L}$

$h = \frac{\vec{j} \cdot \vec{k}}{|\vec{k}|} \to -h$

Op. discretos preservan vacío

$\mathcal{P}|0\rangle = |0\rangle$

KG: $\mathcal{P} a(\vec{p}) \mathcal{P}^{-1} = \eta_p \, a(-\vec{p})$ con $\eta_p^2 = 1$ $\mathcal{P} a^\dagger(p) \mathcal{P}^{-1} = a^\dagger(-p)$ $\mathcal{P}|p\rangle = |-p\rangle$

D: $\mathcal{P} \psi \mathcal{P}^{-1} = \eta_p \gamma^0 \psi(Px)$

$\mathcal{P} \bar{\psi} \mathcal{P}^{-1} = \eta_p^* \bar{\psi}(Px) \gamma^0$

EM: $\mathcal{P} A_\mu(x) \mathcal{P}^{-1} = \mathcal{P}_\mu A_\mu(Px)$

$\boxed{\mathcal{C}}$: $\mathcal{C}|\alpha, \vec{p}, r\rangle = \eta_c |-\alpha, \vec{p}, r\rangle$ (cambia las cargas)

KG: $\mathcal{C} a(\vec{k}) \mathcal{C}^{-1} = \eta_c \, b(\vec{k})$, ...

$\mathcal{C} \phi(x) \mathcal{C}^{-1} = \eta_c \phi^*(x)$

D: $\mathcal{C} \psi(x) \mathcal{C}^{-1} = \psi_c(x) = \eta_c C \bar{\psi}(x)^T$ actúa sobre espacio Dirac ≠ Lorentz (C no cambia x^μ)

$C^{-1} \gamma^\mu C = -\gamma^{\mu T}$; $C^T = -C$; $C = -C^{-1}$ p. ej: $C = i\gamma^2\gamma^0$ (depende de la realiz.)

$\mathcal{C} \bar{\psi} \mathcal{C}^{-1} = \eta_c^* \psi^T C$

EM: $\mathcal{C} A_\mu(x) \mathcal{C}^{-1} = -A_\mu(x)$

$\boxed{\mathcal{T}}$: $x^\mu \to Tx$ $\vec{L}, \vec{S}, \vec{J} \to -\vec{L}, -\vec{S}, -\vec{J}$

$\dot{\vec{x}} \to -\dot{\vec{x}}$ $h \to h$

$\vec{k} \to -\vec{k}$

KG: $\mathcal{T} \phi(x) \mathcal{T}^{-1} = \eta_T \phi(Tx)$ $|\eta_T|^2 = 1$, $\eta_T \in \mathbb{R}$ si $\phi \in \mathbb{R}$ (escalar neutro)

D: $\mathcal{T} \bar{\psi} \mathcal{T}^{-1} = \eta_T^* \bar{\psi}(Tx) T^{-1}$ ... espacio Lorentz

$T \gamma^{\mu T} T^{-1} = \gamma^\mu$; $T = -T^{-1} = -T^T$ ... esp. Dirac p. ej: $T = -C\gamma^5$

Bilineales de Dirac: $\bar{\psi} \Gamma_i \psi$

Tipo	Γ_i	$\mathcal{T}$	$\mathcal{C}$	$\mathcal{P}$	$\Theta = \mathcal{PCT}$
S	I	I	I	I	I
PS	$i\gamma_5$	$-PS$	PS	$-PS$	PS
V	γ^μ	$(V^0, -\vec{V})$	$-V$	$(V^0, -\vec{V})$	$-V$ → $N=1$
VA	$\gamma^\mu\gamma^5$	$(VA^0, -\vec{VA})$	VA	$(-VA^0, \vec{VA})$	$-VA$ → $N=1$
T	$\sigma^{\mu\nu}$	$T_{0i}, -T_{ij}$	$-T$	$-T_{0i}, T_{ij}$	T → $N=2$
Argumentos son:		(Tx)	x	(Px)	$(-x)$

$\Theta \bar{\psi} \Gamma \psi \Theta^{-1} = (-1)^N \bar{\psi}(-x) \Gamma \psi(-x)$

N: nº índices Lorentz

Teorema PCT

- Hermiticidad
- Invariancia Lorentz
- Conexión espín-estadística

$\Longrightarrow$ Invariancia PCT

Θ es simetría de la teoría

→ $m_p = m_{\bar{p}}$ p = partícula, $\bar{p}$ = anti "

→ $\tau_p = \tau_{\bar{p}}$